Internet Appendix for "Cost of Experimentation and the Evolution of Venture Capital"

I. Matching between Entrepreneurs and Investors

No Commitment

Using backward induction we start with the second period and first consider the case when the investor chooses not to commit. Conditional on a given α_1 the investor will invest in the second period as long as

$$V_j \alpha_j E[p_2 \mid j] - (1 - X) > 0 \quad \textit{where } j \in \{S, F\}$$

This condition does not hold after failure even if $\alpha_F = 1$, therefore the investor will only invest after success in the first period. The minimum fraction the investor is willing to accept for an investment of $1 - X$ in the second period after success in the first period is

$$\underline{\alpha_{2S}} = \frac{1 - X}{V_S E[p_2 \mid S]}.$$

The entrepreneur, on the other hand, will continue with the business in the second period as long as,

$$V_j (1 - \alpha_j) E[p_2 \mid j] + u_E > u_F \quad \textit{where } j \in \{S, F\}.$$

The entrepreneur will want to continue if the expected value from continuing is greater than the utility after failure, because the utility after failure is the outside option of the entrepreneur if she does not continue. The maximum fraction the entrepreneur will give up in the second period after success in the first period is

$$\overline{\alpha_{2S}} = 1 - \frac{u_F - u_E}{V_S E[p_2 \mid S]}.$$

Given both the minimum fraction the investor will accept, $\underline{\alpha_{2S}}$, as well as the maximum fraction the entrepreneur will give up, $\overline{\alpha_{2S}}$, an agreement may not be reached. An investor and entrepreneur are able to reach an agreement in the second period as long as

$$1 \geq \underline{\alpha_{2S}} \leq \overline{\alpha_{2S}} \geq 0 \qquad \textit{Agreement Conditions}, \;\; 2^{nd} \textit{period}$$

The middle inequality requirement is that there are gains from trade. However, those gains must also occur in a region that is feasible, i.e. the investor requires less than 100% ownership to be willing to invest, $1 \geq \underline{\alpha_{2S}}$, and the entrepreneur requires less than 100% ownership to be willing to continue, $\overline{\alpha_{2S}} \geq 0$.[1]

We could find the maximum fraction the entrepreneur would be willing to give up after failure ($\overline{\alpha_{2F}}$), however, we already determined that the investor would require a share

[1]If not, the entrepreneur, for example, might be willing to give up 110% of the final payoff and the investor might be willing to invest to get this payoff, but it is clearly not economically feasible. For the same reason, even when there are gains from trade in the reasonable range, the resulting negotiation must yield a fraction such that $0 \leq \alpha_{2j} \leq 1$ otherwise it is bounded by 0 or 1.

($\underline{\alpha_{2F}}$) greater than 100% to invest in the second period, which is not economically viable. So no deal will be done after failure. If an agreement cannot be reached even after success then clearly the deal will never be funded. However, even those projects for which an agreement could be reached after success may not be funded in the first period if the probability of success in the first period is too low. The following proposition determines the conditions for a potential agreement to be reached to fund the project in the first period. Given that the investor can forecast the second period dilution these conditions can be written in terms of the final fraction of the business the investor or entrepreneur needs to own in the successful state in order to be willing to start.

PROPOSITION 1: *The minimum total fraction the investor must receive is*

$$\underline{\alpha_{S_N}} = \frac{p_1(1-X)+X}{p_1 V_S E[p_2 \mid S]}$$

and the maximum total fraction the entrepreneur is willing to give up is

$$\overline{\alpha_{S_N}} = 1 - \frac{(1+p_1)(u_O-u_E)+(1-p_1)(u_O-u_F)}{p_1 V_S E[p_2 \mid S]}$$

where the N subscript represents the fact that no agreement will be reached after failure.

See appendix .IV for proof. We use the N subscript because in the next section we consider the situation when investor chooses to commit to invest in the second period. This will result in an agreement to continue even after first period failure (A subscript for Agreement rather than N for No-agreement). Then we will compare the deals funded in each case. Given the second period fractions found above, the minimum and maximum total fractions imply minimum and maximum first period fractions (found in the appendix for the interested reader).

Commitment

In this subsection we examine the alternative choice by an investor to commit with an assumed cost of early shutdown of c.

The following proposition solves for the minimum fraction the committed investor will accept in the second period and the maximum fraction the entrepreneur will give up in the second period. These will be used to determine if a deal can be reached.

PROPOSITION 2: *The minimum fraction the committed investor is willing to accept for an investment of* $1-X$ *in the second period after success in the first period is*

$$\underline{\alpha_{2S}} = \frac{1-X}{V_S E[p_2 \mid S]}.$$

However, after failure in the first period the minimum fraction the committed investor is willing to accept is

$$\underline{\alpha_{2F}} = \frac{1-X-c}{V_F E[p_2 \mid F](1-\alpha_1)} - \frac{\alpha_1}{1-\alpha_1}.$$

The maximum fraction the entrepreneur will give up in the second period after success in the first period is

$$\overline{\alpha_{2S}} = 1 - \frac{u_F - u_E}{V_S E[p_2 \mid S]}.$$

After failure in the first period the maximum fraction the entrepreneur is willing to give up is

$$\overline{\alpha_{2F}} = 1 - \frac{u_F - u_E}{V_F E[p_2 \mid F](1 - \alpha_1)}.$$

The proof is in appendix .III. Both the investor and the entrepreneur must keep a large enough fraction in the second period to be willing to do a deal rather than choose their outside option. These fractions of course depend on whether or not the first period experiment worked.

After success in the first period the agreement conditions are always met. However, after failure in the first period the agreement conditions may or may not be met depending on the parameters of the investment, the investor and the entrepreneur.

LEMMA 1: *An agreement can be reached in the second period after failure in the first iff the investor is committed.*

PROOF:

A second period deal after failure can be reached if $\overline{\alpha_{2F}} - \underline{\alpha_{2F}} \geq 0$.

$$\overline{\alpha_{2F}} - \underline{\alpha_{2F}} = 1 - \frac{u_F - u_E}{V_F E[p_2 \mid F](1 - \alpha_1)} - \frac{1 - X - c}{V_F E[p_2 \mid F](1 - \alpha_1)} - \frac{\alpha_1}{1 - \alpha_1}.$$

$\overline{\alpha_{2F}} - \underline{\alpha_{2F}}$ is positive iff $V_F E[p_2 \mid F] - u_F + u_E - 1 - X + c \geq 0$. However, since the utility of the entrepreneur cannot be transferred to the investor, it must also be the case that $V_F E[p_2 \mid F] - (1 - X) + c \geq 0$. But if $V_F E[p_2 \mid F] - (1 - X) + c \geq 0$ then $V_F E[p_2 \mid F] - u_F + u_E - (1 - X) + c \geq 0$ because $u_F - u_E < 0$. QED

This lemma makes it clear that only a 'committed' investor will continue to fund the company after failure because $V_F E[p_2 \mid F] - (1 - X) < 0$.

We have now solved for both the minimum second period fraction the committed investor will accept, $\underline{\alpha_{2j}}$, as well as the maximum second period fraction the entrepreneur will give up, $\overline{\alpha_{2j}}$, and the conditions under which a second period deal will be done. If either party yields more than these fractions, then they would be better off accepting their outside, low-risk, opportunity rather than continuing the project in the second period.

Stepping back to the first period, a committed investor will invest and an entrepreneur will start the project with a committed investor only if they expect to end up with a large enough fraction after both first and second period negotiations.

PROPOSITION 3: *The minimum total fraction the investor is willing to accept is*

$$\underline{\alpha_{S_A}} = \frac{1 - (1 - p_1) V_F \alpha_F E[p_2 \mid F]}{p_1 V_S E[p_2 \mid S]},$$

and the maximum fraction the entrepreneur is willing to give up is

$$\overline{\alpha_{S_A}} = 1 - \frac{2\Delta w_1 - (1 - p_1) E[p_2 \mid F] V_F (1 - \alpha_F)}{p_1 V_S E[p_2 \mid S]}$$

where the subscript A signifies that an agreement will be reached after first period failure. And where

$$\alpha_F = \gamma \left[\frac{1 - X - c}{V_F E[p_2 \mid F]} \right] + (1-\gamma) \left[1 - \frac{\Delta u_F}{V_F E[p_2 \mid F]} \right]$$

The proof is in .IV, however, these are the relatively intuitive outcomes in each situation because each player must expect to make in the good state an amount that at least equals their expected cost plus their expected loss in the bad state.

Given the minimum and maximum fractions, we know the project will be started if

$$1 \geq \underline{\alpha_{S_i}} \leq \overline{\alpha_{S_i}} \geq 0 \qquad \textit{Agreement Conditions}, \;\; 1^{st}\ \textit{period},$$

either with our without a second period agreement after failure ($i \in [A, N]$).

We have now calculated the minimum and maximum required by investors and entrepreneurs. With these fractions we can determine the types of projects for which investors will choose to commit.

II. Commitment or the Guillotine

A deal can be done to begin the project if $\underline{\alpha_{S_A}} \leq \overline{\alpha_{S_A}}$, if the investor commits. Alternatively, a deal can be done to begin the project if $\underline{\alpha_{S_N}} \leq \overline{\alpha_{S_N}}$, assuming the project will be shut down after early failure. That is, a deal can get done if the lowest fraction the investor will accept, $\underline{\alpha_{S_i}}$ is less than the highest fraction the entrepreneur with give up, $\overline{\alpha_{S_i}}$. Therefore, given the decision by the investor to commit, a project can be started if $\overline{\alpha_{S_A}} - \underline{\alpha_{S_A}} \geq 0$, i.e., if

$$p_1 V_S E[p_2 \mid S] + (1-p_1) V_F E[p_2 \mid F] - 2(u_O - u_E) - 1 \geq 0, \tag{A-1}$$

or if $\overline{\alpha_{S_N}} - \underline{\alpha_{S_N}} \geq 0$, i.e., if

$$p_1 V_S E[p_2 \mid S] - 2(u_O - u_E) + (1-p_1)\Delta u_F - p_1(1-X) - X \geq 0. \tag{A-2}$$

If we assume that the investor who generates the most surplus wins the deal then an investor will commit if $\overline{\alpha_{S_A}} - \underline{\alpha_{S_A}} \geq \overline{\alpha_{S_N}} - \underline{\alpha_{S_N}}$. Therefore, the following proposition demonstrates the three possibilities for any given project.

The proof is left to Appendix .VI. Proposition 1 demonstrates the potential for a tradeoff between failure tolerance and the funding of a new venture. There is both a benefit of a sharp guillotine as well as a cost. Entrepreneurs do not like to be terminated 'early' and thus would rather receive and investment from a committed investor. This failure tolerance encourages innovative effort (as in **???**), but a committed investor gives up the valuable real option to terminate the project early. Thus, the committed investor must receive a larger fraction of the pie if successful. While the entrepreneur would like a committed investor the commitment comes at a price. For some projects and entrepreneurs that price is so high that they would rather not do the deal. For others they would rather do the deal, but just not with a committed investor.

Essentially the utility of the entrepreneur can be enhanced by moving some of the payout in the success state to the early failure state. This is accomplished by giving a more failure tolerant VC a larger initial fraction in exchange for the commitment to fund the project in the bad state. If the entrepreneur is willing to pay enough in the good state to the investor to make that trade worth it to the investor then the deal can be done. However, there are deals for which this is true and deals for which this is not true. If the committed investor requires too much in order to be failure tolerant in the bad state, then the deal may be done by a VC with a sharp guillotine.

III. *Proof of Proposition 2*

Conditional on a given α_1 the investor will invest in the second period as long as

$$V_j \alpha_j E[p_2 \mid j] - (1-X) > -c \quad \textit{where } j \in \{S, F\}$$

As noted above, c, is the cost faced by the investor when he stops funding a project and it dies. Thus, the minimum fraction the investor will accept in the second period is

$$\underline{\alpha_{2j}} = \frac{(1-x)-c}{V_j E[p_2 \mid j](1-\alpha_1)} - \frac{\alpha_1}{1-\alpha_1}.$$

Thus, an investor will not invest in the second period unless the project is NPV positive accounting for the cost of shutdown. This suggests that an investor who already owned a fraction of the business, α_1, from the first period would be willing to take a lower minimum fraction in the second period than a new investor, and potentially accept even a negative fraction. However, there is a fraction η such that the investor is better off letting an outside investor invest (as long as an outside investor is willing to invest) rather that accept a smaller fraction. If $V_j E[p_2 \mid j] > (1-X)$ (which is true for $j = S$) then an outside investor would invest for a fraction greater than or equal to $\frac{1-X}{V_S E[p_2|S]}$. The fraction η that makes the investor indifferent between investing or not is the η such that

$$\alpha_1(1-\eta)V_S E[p_2 \mid S]) = (\eta + \alpha_1(1-\eta))V_S E[p_2 \mid S] - (1-X)$$

The left hand side is what the first period investor expects if a new investor purchases η in the second period. While the right hand side is the amount the first period investor expects if he purchases η in the second period. The η that makes this equality hold is $\eta = \frac{1-X}{V_S E[p_2|S]}$. Note that η does not depend on c because the project continues either way. Thus, after success, an old investor is better off letting a new investor invest than accepting a fraction less than $\frac{1-X}{V_S E[p_2|S]}$.[2] Thus, the correct minimum fraction that the investor will accept for an investment of $1-X$ in the second period after success in the first period is

$$\underline{\alpha_{2S}} = \frac{1-X}{V_S E[p_2 \mid S]}.$$

However, after failure in the first period then $V_F E[p_2 \mid F] < 1-X$ and no new investor will invest. Potentially an old (committed) investor would still invest (to avoid paying c)

[2] This assumes perfect capital markets that would allow a 'switching' of investors if entrepreneurs tried to extract too much. No results depend on this assumption but it makes the math easier and more intuitive, and we don't want to drive any results off of financial market frictions.

and the minimum fraction he would accept is

$$\underline{\alpha_{2F}} = \frac{1 - X - c}{V_F E[p_2 \mid F](1-\alpha_1)} - \frac{\alpha_1}{1-\alpha_1}.$$

The entrepreneur, on the other hand, will continue with the business in the second period as long as,

$$V_j(1-\alpha_j)E[p_2 \mid j] + u_E > u_F \quad where\ j \in \{S, F\}.$$

Since $\alpha_j = \alpha_{2j} + \alpha_1(1-\alpha_{2j})$, for a given α_1 the maximum fraction the entrepreneur will give to the investor in the second period is

$$\overline{\alpha_{2j}} = 1 - \frac{u_F - u_E}{V_j E[p_2 \mid j](1-\alpha_1)} \quad \forall\ j \in \{S, F\}.$$

Similarly to the investor, after success in the first period, there is a point at which the entrepreneur who already owns a fraction $1-\alpha_1$ should quit and let the investors hire a new manager rather than take a smaller fraction. Thus, there is a η that makes the entrepreneur indifferent between staying and leaving:

$$(1-\alpha_1)\eta V_S E[p_2 \mid S] + u_F = ((1-\eta) + (1-\alpha_1)\eta)V_S E[p_2 \mid S] + u_E$$

Thus, the correct maximum fraction the entrepreneur will give up in the second period after success in the first period is[3]

$$\overline{\alpha_{2S}} = 1 - \frac{u_F - u_E}{V_S E[p_2 \mid S]}$$

However, after failure in the first period the maximum that the entrepreneur is willing to give up to keep the business alive is

$$\overline{\alpha_{2F}} = 1 - \frac{u_F - u_E}{V_F E[p_2 \mid F](1-\alpha_1)}$$

The entrepreneur cannot credibly threaten to leave after failure unless he must give up more than $\overline{\alpha_{2F}}$, as his departure will just cause the business to be shut down.

IV. Proof of Propositions 1 and 3

Bargaining will result in a fraction in the second period of $\alpha_{2j} = \gamma\underline{\alpha_{2j}} + (1-\gamma)\overline{\alpha_{2j}}$. For example, if the entrepreneur has all the bargaining power, $\gamma = 1$, then the investor must accept his minimum fraction, $\alpha_{2j} = \underline{\alpha_{2j}}$, while if the investor has all the bargaining power, $\gamma = 0$, then the entrepreneur must give up the maximum, $\alpha_{2j} = \overline{\alpha_{2j}}$. While if each has some bargaining power then they share the surplus created by the opportunity.

Given this, we can substitute into $\alpha_j = \alpha_{2j} + \alpha_1(1-\alpha_{2j})$ and solve for the final fractions the investor and entrepreneur will obtain depending on success or failure at the first stage.

[3]This requires the assumption of perfect labor markets that would allow a 'switching' of CEOs among entrepreneurial firms if investors tried to extract too much. No results depend on this assumption but it makes the math easier and more intuitive, and we don't want to drive any results off of labor market frictions.

Substituting we find $\alpha_j = \gamma\underline{\alpha_{2j}} + (1-\gamma)\overline{\alpha_{2j}} + \alpha_1(1-(\gamma\underline{\alpha_{2j}} + (1-\gamma)\overline{\alpha_{2j}}))$. This can be rewritten as $\alpha_j = [\gamma\underline{\alpha_{2j}} + (1-\gamma)\overline{\alpha_{2j}}](1-\alpha_1)+\alpha_1$. Substituting in for $\underline{\alpha_{2j}}$ and $\overline{\alpha_{2j}}$ we find that

$$\alpha_S = \left[\gamma\frac{1-X}{V_S E[p_2 \mid S]} + (1-\gamma)\left[1 - \frac{u_F - u_E}{V_S E[p_2 \mid S]}\right]\right](1-\alpha_1)+\alpha_1 \tag{A-3}$$

and α_F reduces to

$$\alpha_F = \gamma\left[\frac{1-X-c}{V_F E[p_2 \mid F]}\right] + (1-\gamma)\left[1-\frac{u_F-u_E}{V_F E[p_2 \mid F]}\right] \tag{A-4}$$

Of course, in both cases negotiations must result in a fraction between zero and one.[4] Note that α_F does not depend on the negotiations in the first period because after failure, renegotiation determines the final fractions.[5] Of course, investors and entrepreneurs will account for this in the first period when they decide whether or not to participate.[6] We solve for the first period fractions in appendix .V but these are not necessary for the proof.

The solution α_F is only correct *assuming* a deal can be reached between the investor and the entrepreneur in the second period (otherwise the company is shut down after early failure). Interesting outcomes will emerge both when an agreement can and cannot be reached as this will affect both the price of, and the willingness to begin, a project.

Stepping back to the first period, an investor will invest as long as

$$p_1[V_S\alpha_S E[p_2 \mid S] - (1-X)] - X + (1-p_1)[V_F\alpha_F E[p_2 \mid F] - (1-X)] \geq 0 \tag{A-5}$$

if the 2nd period agreement conditions are met after failure. Or,

$$p_1[V_S\alpha_S E[p_2 \mid S] - (1-X)] - X - (1-p_1)c \geq 0 \tag{A-6}$$

if they are not.

The entrepreneur will choose to innovate and start the project if

$$p_1[V_S(1-\alpha_S)E[p_2 \mid S] + u_E] + u_E + (1-p_1)[V_F(1-\alpha_F)E[p_2 \mid F] + u_E] \geq 2u_O \tag{A-7}$$

if the 2nd period agreement conditions are met after failure. Or,

$$p_1[V_S(1-\alpha_S)E[p_2 \mid S] + u_E] + u_E + (1-p_1)u_F \geq 2u_O \tag{A-8}$$

if they are not.

The four above equations can be used to solve for the minimum fractions needed by

[4]Since negotiations must result in a fraction between zero and one, then if a deal can be done then if $\gamma < (u_F - u_E)/(Y(1+r) - c - V_F E[p_2 \mid F] + u_F - u_E)$ then $\alpha_F = 1$, or if $\gamma < -(u_F - u_E)/(Y(1+r) - V_S E[p_2 \mid S] + u_F - u_E)$ then $\alpha_S = 1$. Since $c \leq 1 - X$ the negotiations will never result in a fraction less than zero.

[5]In actual venture capital deals so called 'down rounds' that occur after poor outcomes often result in a complete rearrangement of ownership fractions between the first round, second round and entrepreneur.

[6]Alternatively we could assume that investors and entrepreneurs predetermine a split for for every first stage outcome. This would require complete contracts and verifiable states so seems less realistic but would not change the intuition or implications of our results.

the investor and entrepreneur both when a deal after failure can be reached and when it cannot. If the agreement conditions in the 2nd period after failure are met, then the minimum fraction the investor is willing to receive in the successful state and still choose to invest in the project is found by solving equation (A-5) for the minimum α_S such that the inequality holds:

$$\underline{\alpha_{S_A}} = \frac{1-(1-p_1)V_F\alpha_F E[p_2 \mid F]}{p_1 V_S E[p_2 \mid S]}$$

where the subscript A signifies that an agreement can be reached after first period failure. The maximum fraction the entrepreneur can give up in the successful state and still be willing to choose the entrepreneurial project is found by solving equation (A-7) for the maximum α_S such that the inequality holds:

$$\overline{\alpha_{S_A}} = 1 - \frac{2(u_O-u_E)-(1-p_1)E[p_2 \mid F]V_F(1-\alpha_F)}{p_1 V_S E[p_2 \mid S]}$$

where α_F is defined in equation (A-4) in both $\overline{\alpha_{S_A}}$ and $\underline{\alpha_{S_A}}$. Both $\overline{\alpha_{S_A}}$ and $\underline{\alpha_{S_A}}$ depend on the negotiations in the failed state, α_F, because the minimum share the players need to receive in the the good state to make them willing to choose the project depends on how badly they do in the bad state. If a second period agreement after failure cannot be reached then the minimum fraction of the investor and the maximum fraction of the entrepreneur are found by solving equations (A-6) and (A-8) respectively, to find

$$\underline{\alpha_{S_N}} = \frac{p_1(1-X)+X}{p_1 V_S E[p_2 \mid S]}$$

and

$$\overline{\alpha_{S_N}} = 1 - \frac{(1+p_1)(u_O-u_E)+(1-p_1)(u_O-u_F)}{p_1 V_S E[p_2 \mid S]}$$

where the N subscript represents the fact that no agreement can be reached after failure.

V. *Derivation of first period fractions*

The maximum and minimum required shares after first period success, $\overline{\alpha_{S_i}}$ and $\underline{\alpha_{S_i}}$, directly imply first period minimum an maximum fractions, $\overline{\alpha_{1_i}}$ and $\underline{\alpha_{1_i}}$ ($i \in [A, N]$), because we already know from above, equation (A-3), that

$$\alpha_S = \left[\gamma\frac{1-X}{V_S E[p_2 \mid S]} + (1-\gamma)(1-\frac{u_F-u_E}{V_S E[p_2 \mid S]})\right](1-\alpha_1)+\alpha_1$$

Thus, we can solve for the α_1 that just gives the investor his minimum α_S. Let Z equal the term in brackets in the equation above and we can solve for α_1 as a function of α_S.

$$\alpha_1 = \frac{\alpha_S - Z}{1-Z} \tag{A-9}$$

Plugging in $\underline{\alpha_{S_A}}$ for α_S yields the minimum required investor fraction $\underline{\alpha_{1_A}}$:

$$\underline{\alpha_{1_A}} = \frac{\frac{1-(1-p_1)V_F\alpha_F E[p_2|F]}{p_1 V_S E[p_2|S]} - Z}{1-Z}$$

as a function of α_F. And substituting in for α_F from equation (A-4) and Z from above yields,

$$\underline{\alpha_{1_A}} = 1 - \frac{p_1 V_S E[p_2 \mid S] - p_1(1-X) - X - (1-p_1)\gamma c}{p_1(\gamma V_S E[p_2 \mid S] - \gamma(1-X) + (1-\gamma)(u_F - u_E))} - \frac{(1-p_1)(1-\gamma)(V_F E[p_2 \mid F] - (1-X) - (u_F - u_E))}{p_1(\gamma V_S E[p_2 \mid S] - \gamma(1-X) + (1-\gamma)(u_F - u_E))}$$

This is the minimum fraction required by the investor assuming that a deal can be achieved in the second period after failure in the first period.[7] In equilibrium the investor's minimum depends on the entrepreneur's gains and costs because they must negotiate and participate. If instead, an agreement cannot be reached after failure in the first period then the project is stopped. In this case the minimum fraction required by the investor can be found by plugging $\underline{\alpha_{S_N}}$ into equation (A-9) for α_S, where $\underline{\alpha_{S_N}}$ is the minimum when no second period deal can be reached. In this case the minimum required investor fraction $\underline{\alpha_{1_N}}$ is

$$\underline{\alpha_{1_N}} = \frac{\frac{p_1(1-X)+X}{p_1 V_S E[p_2|S]} - Z}{1-Z}$$

or,

$$\underline{\alpha_{1_N}} = 1 - \frac{p_1 V_S E[p_2 \mid S] - p_1(1-X) - X}{p_1(\gamma V_S E[p_2 \mid S] - \gamma(1-X) + (1-\gamma)(u_F - u_E))}$$

We can similarly calculate the maximum fraction the entrepreneur is willing to give up in the first period. The maximum fraction can be found by plugging $\overline{\alpha_{S_i}}$ into equation (A-9) for α_{S_i}, where $\overline{\alpha_{S_i}}$ $(i \in [A, N])$ is the maximum when either a second period agreement after failure can (A) or cannot (N) be reached. When a second period agreement can be reached $\overline{\alpha_{1_A}}$ is

$$\overline{\alpha_{1_A}} = 1 - \frac{2(u_O - u_E) - (1-p_1)E[p_2 \mid F]V_F(1-\alpha_F)}{p_1(\gamma V_S E[p_2 \mid S] - \gamma Y(1+r) + (1-\gamma)(u_F - u_E))}$$

And when a second period deal after failure cannot be reached $\overline{\alpha_{1_N}}$ is

$$\overline{\alpha_{1_N}} = 1 - \frac{(1+p_1)(u_O - u_E) + (1-p_1)(u_O - u_F)}{p_1(\gamma V_S E[p_2 \mid S] - \gamma(1-X) + (1-\gamma)(u_F - u_E))}$$

[7]Technical note: with extreme values it is possible that α_F would be greater than 1 or less than zero. In these cases α_F is bound by either zero or 1. This would cause the $\underline{\alpha_1}$ to increase or decrease. This dampens some of the effects in extreme cases but alters no results. To simplify the exposition we assume that parameters are in the reasonable range such that the investor and entrepreneur would not be willing to agree to a share greater than 1 or less than zero.

VI. *Proof of Proposition 1:*

It is clearly possible that both $\overline{\alpha_{S_A}} - \underline{\alpha_{S_A}} < 0$ and $\overline{\alpha_{S_N}} - \underline{\alpha_{S_N}} < 0$. For example, a project with a low enough V_S and/or V_F could have both differences less than zero. Similarly, for a high enough V_S and/or V_F (or low X) both $\overline{\alpha_{S_A}} - \underline{\alpha_{S_A}} > 0$ and $\overline{\alpha_{S_N}} - \underline{\alpha_{S_N}} > 0$, even for c equal to the maximum c of $1 - X$. Thus, extremely bad projects will not be started and extremely good projects may be started by any type of investor.

If either $\overline{\alpha_{S_A}} - \underline{\alpha_{S_A}} \geq 0$ or $\overline{\alpha_{S_N}} - \underline{\alpha_{S_N}} \geq 0$ or both then the investor will generate more surplus by committing if $\overline{\alpha_{S_A}} - \underline{\alpha_{S_A}} \geq \overline{\alpha_{S_N}} - \underline{\alpha_{S_N}}$ or vice versa. The difference between $\overline{\alpha_{S_A}} - \underline{\alpha_{S_A}}$ and $\overline{\alpha_{S_N}} - \underline{\alpha_{S_N}}$ is

$$\frac{(1-p_1)V_F E[p_2 \mid F] - (1-p_1)\Delta u_F - (1-p_1)(1-X)}{p_1 V_S E[p_2 \mid S]} \tag{A-10}$$

Equation (A-10) may be positive or negative depending on the relative magnitudes of $V_F E[p_2 \mid F]$, Δu_F, and $(1 - X)$. That is, projects for which the first stage experiment is cheap (X is small) and the utility impact on the entrepreneur from shutting down the project is low (Δu_F is small) and the expected value after failure is low ($V_F E[p_2 \mid F]$ is small) are more likely to be done by an uncommitted investor. QED

VII. *Proof of Proposition 2 and Corollary 1:*

From above we know that when Equation (A-10) is greater than zero then $\overline{\alpha_{S_A}} - \underline{\alpha_{S_A}} \geq \overline{\alpha_{S_N}} - \underline{\alpha_{S_N}}$ and the project creates more value if funded by a committed investor. This is more likely if $V_F E[p_2 \mid F]$ is larger, Δu_F is smaller, or $(1 - X)$ is smaller.

The Corollary follows directly from the fact that if two projects have the same expected value, $p_1 V_S E[p_2 \mid S] + (1-p_1) V_F E[p_2 \mid F]$, and same probability of a successful experiment, p_1, but a more valuable experiment ($V_S E[p_2 \mid S] - V_F E[p_2 \mid F]$ is larger) then $V_F E[p_2 \mid F]$ must be smaller. QED

VIII. *Proof of Proposition 3:*

A project will be funded by a committed investor if $\overline{\alpha_{S_A}} - \underline{\alpha_{S_A}} \geq \overline{\alpha_{S_N}} - \underline{\alpha_{S_N}}$ or

$$(1-p_1)V_F E[p_2 \mid F] - (1-p_1)\Delta u_F - (1-p_1)(1-X) \geq 0 \tag{A-11}$$

The derivative of this condition with respect to X is $(1 - p_1)$. Thus a firm is more likely to switch type of funder with a fall in X if it has a small probability of first period success, p_1. Furthermore, if X falls then $(1 - p_1)(1 - X)$ is larger and it takes a larger $V_F E[p_2 \mid F]$ for a committer to win. Thus, the projects that switch will be those with lower $V_F E[p_2 \mid F]$. For a given expected value if $V_F E[p_2 \mid F]$ is smaller then $V_S E[p_2 \mid S]$ must be larger and $V_S E[p_2 \mid S] - V_F E[p_2 \mid F]$ is larger so the project has a more valuable experiment. A project will be funded by an uncommitted investor rather than no investor if

$$p_1 V_S E[p_2 \mid S] - 2(u_O - u_E) + (1-p_1)\Delta u_F - p_1(1-X) - X \geq 0. \tag{A-12}$$

The derivative of this condition with respect to X is $p_1 - 1$. Therefore an increase in X has a larger (more negative) impact if p_1 is small. These firms have a smaller $V_S E[p_2 \mid S]$

than those funded by an uncommitted investor before the change in X. However, before the change a committed investor would have funded this set of firms if they had a higher $V_F E[p_2 \mid F]$. This can be seen by noting that committed investors are willing to fund a project if $\overline{\alpha_{S_A}} - \underline{\alpha_{S_A}} \geq 0$, i.e., if

$$p_1 V_S E[p_2 \mid S] + (1 - p_1) V_F E[p_2 \mid F] - 2(u_O - u_E) - 1 \geq 0, \tag{A-13}$$

QED